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On One-Parameter Supercoherent State of spl(2,1) Superalgebra

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Abstract One-Parameter supercoherent state of the spl$(2,1)$ superalgebra is constructed and its properties are discussed in detail. The parameter α may be related to the interaction parameter U in one exactly solvable model for correlated electrons.

Keywords spl$(2,1)$ superalgebra · Supercoherent state · Exactly solvable model

1 Introduction

One-parameter irreducible representations of Lie superalgebra have played an important role in constructing supersymmetrical models. The supersymmetrical algebra of BGLZ model for correlated electrons on the unrestricted 4^L-dimensional electronic Hilbert space $\bigotimes_{n=1}^{L} C^4$ is superalgebra gl$(2|1)$ [1]. It is interesting that those models contain one symmetry-preserving free real parameter which is the Hubbard interaction parameter U. The coherent states of Lie (super)algebras are very important in the study of quantum mechanics, quantum electrodynamics, quantum optics and quantum field theory, which provide a natural link between classical and quantum phenomena and are related to the path integral formalism. One-parameter indecomposable and irreducible representations of the spl$(2,1)$ superalgebra have been studied [2, 3]. The purpose of the present paper is to derive further the new supercoherent state of the spl$(2,1)$ superalgebra on the basis of studying one-parameter irreducible representation, and discuss its properties. In the present paper we shall first construct the supercoherent state of the spl$(2,1)$ superalgebra. Then we discuss its properties. In other article, we shall give a new form of the inhomogeneous differential realizations of the spl$(2,1)$ in one-parameter supercoherent-state space.

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2 The spl(2,1) One-Parameter Supercoherent State

In accordance with the [4] the generators of the spl(2,1) superalgebra read as follows:

$$\{Q_3, Q_+, Q_-, B \in \mathrm{spl}(2,1)_{\bar{0}} | V_+, V_-, W_+, W_- \in \mathrm{spl}(2,1)_{\bar{1}}\} \tag{1}$$

and satisfy the following commutation and anticommutation relations:

$$\begin{aligned}
&[Q_3, Q_\pm] = \pm Q_\pm, \qquad [Q_+, Q_-] = 2Q_3, \qquad [B, Q_\pm] = [B, Q_3] = 0,\\
&[Q_3, V_\pm] = \pm\frac{1}{2}V_\pm, \qquad [Q_3, W_\pm] = \pm\frac{1}{2}W_\pm, \qquad [B, V_\pm] = \frac{1}{2}V_\pm,\\
&[B, W_\pm] = -\frac{1}{2}W_\pm, \qquad [Q_\pm, V_\mp] = V_\pm, \qquad [Q_\pm, W_\mp] = W_\pm, \qquad [Q_\pm, V_\pm] = 0,\\
&[Q_\pm, W_\pm] = 0, \qquad \{V_\pm, V_\pm\} = \{V_\pm, V_\mp\} = \{W_\pm, W_\pm\} = \{W_\pm, W_\mp\} = 0,\\
&\{V_\pm, W_\pm\} = \pm Q_\pm, \qquad \{V_\pm, W_\mp\} = -Q_3 \pm B.
\end{aligned} \tag{2}$$

In [5] we gave a typical 4-dimensional one-parameter elementary representation of the spl(2,1)

$$\begin{aligned}
D(Q_3) &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D(Q_+) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\\
D(Q_-) &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D(B) = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \frac{1}{2}+\alpha & 0 & 0 \\ 0 & 0 & \frac{1}{2}+\alpha & 0 \\ 0 & 0 & 0 & 1+\alpha \end{bmatrix}.
\end{aligned} \tag{3}$$

From (3) we have obtained one-parameter indecomposable and irreducible representations of the spl(2,1) superalgebra on the quotient space of V [2, 3]

$$\begin{aligned}
Y = (V/J) : \{&\phi(k, \alpha_1, \alpha_2) = \phi(k, 0, \alpha_1, 0, \alpha_2, 0)\\
&\mathrm{mod}\ J | k \in Z^+, \alpha_1, \alpha_2 = 0, 1\}
\end{aligned}$$

relabelling the basis vector $\phi(k, \alpha_1, \alpha_2)$ of the finite-dimensional irreducible representation of the spl(2,1) superalgebra by $|N, k, \alpha_1, \alpha_2\rangle$ the actions of the generators on the basis vectors are

$$\begin{aligned}
Q_3|N, k, \alpha_1, \alpha_2\rangle &= \left(-\frac{1}{2}N + k + \frac{1}{2}\alpha_1 + \frac{1}{2}\alpha_2\right)|N, k, \alpha_1, \alpha_2\rangle,\\
B|N, k, \alpha_1, \alpha_2\rangle &= \left[\left(\frac{1}{2} + \alpha\right)N - \frac{1}{2}\alpha_1 - \frac{1}{2}\alpha_2\right]|N, k, \alpha_1, \alpha_2\rangle,\\
Q_+|N, k, \alpha_1, \alpha_2\rangle &= (N - k - \alpha_1 - \alpha_2)|N, k+1, \alpha_1, \alpha_2\rangle,\\
Q_-|N, k, \alpha_1, \alpha_2\rangle &= k|N, k-1, \alpha_1, \alpha_2\rangle,\\
V_+|N, k, \alpha_1, \alpha_2\rangle &= \alpha_1\sqrt{\alpha}|N, k+1, \alpha_1 - 1, \alpha_2\rangle\\
&\quad + (-1)^{\alpha_1}(1 - \alpha_2)(N - k - \alpha_1)\sqrt{1+\alpha}|N, k, \alpha_1, \alpha_2 + 1\rangle,
\end{aligned} \tag{4}$$

$$
\begin{aligned}
V_-|N,k,\alpha_1,\alpha_2\rangle &= \alpha_1\sqrt{\alpha}|N,k,\alpha_1-1,\alpha_2\rangle \\
&\quad - (-1)^{\alpha_1}(1-\alpha_2)\sqrt{1+\alpha}k|N,k-1,\alpha_1,\alpha_2+1\rangle, \\
W_+|N,k,\alpha_1,\alpha_2\rangle &= (-1)^{\alpha_1}\alpha_2\sqrt{1+\alpha}|N,k+1,\alpha_1,\alpha_2-1\rangle \\
&\quad + (-N+k+\alpha_2)(1-\alpha_1)\sqrt{\alpha}|N,k,\alpha_1+1,\alpha_2\rangle, \\
W_-|N,k,\alpha_1,\alpha_2\rangle &= (1-\alpha_1)\sqrt{\alpha}k|N,k-1,\alpha_1+1,\alpha_2\rangle \\
&\quad + (-1)^{\alpha_1}\alpha_2\sqrt{1+\alpha}|N,k,\alpha_1,\alpha_2-1\rangle,
\end{aligned}
$$

where $\{|N,k,\alpha_1,\alpha_2\rangle \mid k+\alpha_1+\alpha_2 \le N, N \in Z^+, k=0,1,2,\dots,\alpha_1,\alpha_2=0,1\}$ and

$$
k = \begin{cases} 0,1,\dots,N & \text{when } \alpha_1=0,\alpha_2=0, \\ 0,1,\dots,N-1 & \text{when } \alpha_1=0,\alpha_2=1, \\ 0,1,\dots,N-1 & \text{when } \alpha_1=1,\alpha_2=0, \\ 0,1,\dots,N-2 & \text{when } \alpha_1=1,\alpha_2=1. \end{cases}
$$

The space $\{|N,k,\alpha_1,\alpha_2\rangle\}$ of the irrep N of the spl(2,1) superalgebra is $4N$ dimensional and may be divided into four subspaces $\{|N,k,0,0\rangle\}$, $\{|N,k,0,1\rangle\}$, $\{|N,k,1,0\rangle\}$ and $\{|N,k,1,1\rangle\}$ corresponding to $(\alpha_1,\alpha_2)=(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$, respectively. All the basis vectors $|N,k,\alpha_1,\alpha_2\rangle$ are assumed to be normalized as

$$
\begin{aligned}
&\binom{N}{k}\langle N,k,0,0|N,k,0,0\rangle = 1, \qquad \binom{N-1}{k}\langle N,k,0,1|N,k,0,1\rangle = 1, \\
&\binom{N-1}{k}\langle N,k,1,0|N,k,1,0\rangle = 1, \qquad \binom{N-2}{k}\langle N,k,1,1|N,k,1,1\rangle = 1.
\end{aligned} \tag{5}
$$

The completeness condition of the vectors of the irrep may be expressed as

$$
\begin{aligned}
&\sum_{k=0}^{N}\binom{N}{k}|N,k,0,0\rangle\langle N,k,0,0| + \sum_{k=0}^{N-1}\binom{N-1}{k}|N,k,0,1\rangle\langle N,k,0,1| \\
&\quad + \sum_{k=0}^{N-1}\binom{N-1}{k}|N,k,1,0\rangle\langle N,k,1,0| + \sum_{k=0}^{N-2}\binom{N-2}{k}|N,k,1,1\rangle\langle N,k,1,1| = I,
\end{aligned} \tag{6}
$$

where I is the identity operator.

From (4) we can easily derive the following formulas

$$
\begin{aligned}
Q_+^n|N,0,0,0\rangle &= \binom{N}{n}n!\,|N,n,0,0\rangle, \\
Q_+^n|N,0,0,1\rangle &= \binom{N-1}{n}n!\,|N,n,0,1\rangle, \\
Q_+^n|N,0,1,0\rangle &= \binom{N-1}{n}n!\,|N,n,1,0\rangle, \\
Q_+^n|N,0,1,1\rangle &= \binom{N-2}{n}n!\,|N,n,1,1\rangle,
\end{aligned} \tag{7}
$$

where $\binom{N}{n} = \frac{N!}{(N-n)!n!}$.

Now, we define a one-parameter supercoherent state $|Z,\xi_1,\xi_2\rangle$ by applying the exponential operator $\exp(ZQ_+ + \xi_1 V_+ + \xi_2 W_+)$ on the lowest-weight state $|N,0,0,0\rangle$ of the spl$(2,1)$ irrep

$$|Z,\xi_1,\xi_2\rangle = \exp(ZQ_+ + \xi_1 V_+ + \xi_2 W_+)|N,0,0,0\rangle, \tag{8}$$

where Z,ξ_1 and ξ_2 are one complex variable and two Grassmann variables, respectively. Considering the generator Q_+ being commutable with V_+ and W_+, and anticommutation relation of Grassmann variables ξ_1,ξ_2,

$$\{\xi_1,\xi_2\} = 0 \tag{9}$$

we can easily derive the following formula

$$\begin{aligned}&\exp(ZQ_+ + \xi_1 V_+ + \xi_2 W_+)\\ &\quad= \exp\left(\left(Z - \frac{1}{2}\xi_1\xi_2\right)Q_+\right)\exp(\xi_1 V_+)\exp(\xi_2 W_+).\end{aligned} \tag{10}$$

Using the formulas (7) and (10), the one-parameter supercoherent state (8) may be rewritten as follows:

$$\begin{aligned}|Z,\xi_1,\xi_2\rangle &= \sum_{n=0}^{N}\binom{N}{n}Z^n|N,n,0,0\rangle\\ &\quad+ N\xi_1\sqrt{1+\alpha}\sum_{n=0}^{N-1}\binom{N-1}{n}Z^n|N,n,0,1\rangle\\ &\quad- N\xi_2\sqrt{\alpha}\sum_{n=0}^{N-1}\binom{N-1}{n}Z^n|N,n,1,0\rangle\\ &\quad+ N(N-1)\xi_1\xi_2\sqrt{\alpha(1+\alpha)}\sum_{n=0}^{N-2}\binom{N-2}{n}Z^n|N,n,1,1\rangle.\end{aligned} \tag{11}$$

Let $|Z\rangle_1, |Z\rangle_2, |Z\rangle_3$ and $|Z\rangle_4$ are four simple coherent states associated with four subspaces $\{|N,k,0,0\rangle\}, \{|N,k,0,1\rangle\}, \{|N,k,1,0\rangle\}$ and $\{|N,k,1,1\rangle\}$ of the spl$(2,1)$ irrep, we define

$$\begin{aligned}&|Z\rangle_1 = \sum_{n=0}^{N}\binom{N}{n}Z^n|N,n,0,0\rangle, \qquad |Z\rangle_2 = \sum_{n=0}^{N-1}\binom{N-1}{n}Z^n|N,n,0,0\rangle,\\ &|Z\rangle_3 = \sum_{n=0}^{N-1}\binom{N-1}{n}Z^n|N,n,0,0\rangle, \qquad |Z\rangle_4 = \sum_{n=0}^{N-2}\binom{N-2}{n}Z^n|N,n,0,0\rangle.\end{aligned} \tag{12}$$

Therefore (11) can be simply written as

$$\begin{aligned}|Z,\xi_1,\xi_2\rangle &= |Z\rangle_1 + N\xi_1\sqrt{1+\alpha}|Z\rangle_2 - N\xi_2\sqrt{\alpha}|Z\rangle_3\\ &\quad+ N(N-1)\xi_1\xi_2\sqrt{\alpha(1+\alpha)}|Z\rangle_4.\end{aligned} \tag{13}$$

3 One-Parameter Supercoherent State Properties of the spl(2,1)

According to (13) we have

$$\langle Z,\xi_1,\xi_2| = {}_1\langle Z| + N\bar{\xi}_1\sqrt{1+\alpha}\,{}_2\langle Z| - N\bar{\xi}_2\sqrt{\alpha}\,{}_3\langle Z| \\ + N(N-1)\bar{\xi}_2\bar{\xi}_1\sqrt{\alpha(1+\alpha)}\,{}_4\langle Z|, \tag{14}$$

where $\bar{\xi}_1, \bar{\xi}_2$ are the complex conjugation of ξ_1, ξ_2.

We may write the scalar product of two such states as ${}_i\langle Z'|Z\rangle_i$. We see from (12) and (5) that these scalar products are

$$\begin{aligned} &{}_1\langle Z'|Z\rangle_1 = (1+\bar{Z}'Z)^N, \qquad {}_2\langle Z'|Z\rangle_2 = (1+\bar{Z}'Z)^{N-1}, \\ &{}_3\langle Z'|Z\rangle_3 = (1+\bar{Z}'Z)^{N-1}, \qquad {}_4\langle Z'|Z\rangle_4 = (1+\bar{Z}'Z)^{N-2} \quad \text{and} \\ &{}_i\langle Z'|Z\rangle_j = 0 \quad (i \neq j, i, j = 1,2,3,4) \end{aligned} \tag{15}$$

which means that the two simple coherent states with different Z in the same subspace are not orthogonal to each other. Nevertheless, two coherent states in different subspaces are orthogonal to each other.

Similarly, the scalar product of the one-parameter supercoherent state is written as follows:

$$\begin{aligned} &\langle Z',\xi_1',\xi_2'|Z,\xi_1,\xi_2\rangle \\ &\quad = \{[1+\bar{Z}'Z + N^2((1+\alpha)\bar{\xi}_1'\xi_1 + \alpha\bar{\xi}_2'\xi_2)](1+\bar{Z}'Z) \\ &\qquad + N^2(N-1)^2\alpha(1+\alpha)\bar{\xi}_1'\xi_1\bar{\xi}_2'\xi_2\}(1+\bar{Z}'Z)^{N-2}. \end{aligned} \tag{16}$$

Making $Z' = Z, \xi_1' = \xi_1, \xi_2' = \xi_2$ in (16) we may write the orthogonality relation of the supercoherent state $|Z,\xi_1,\xi_2\rangle$,

$$\begin{aligned} &\langle Z,\xi_1,\xi_2 \mid Z,\xi_1,\xi_2\rangle \\ &\quad = \{[1+\bar{Z}Z + N^2((1+\alpha)\bar{\xi}_1\xi_1 + \alpha\bar{\xi}_2\xi_2)](1+\bar{Z}Z) \\ &\qquad + N^2(N-1)^2\alpha(1+\alpha)\bar{\xi}_1\xi_1\bar{\xi}_2\xi_2\}(1+\bar{Z}Z)^{N-2}. \end{aligned} \tag{17}$$

The expansion coefficients of the supercoherent state $|Z,\xi_1,\xi_2\rangle$ may be found in terms of the complete orthonormal set $\{|N,k,\alpha_1,\alpha_2\rangle\}$. Thus, we have

$$\begin{aligned} &\langle Z,\xi_1,\xi_2|N,k,0,0\rangle = \bar{Z}^k, \\ &\langle Z,\xi_1,\xi_2|N,k,0,1\rangle = N\sqrt{1+\alpha}\,\bar{\xi}_1\bar{Z}^k, \\ &\langle Z,\xi_1,\xi_2|N,k,1,0\rangle = -N\sqrt{\alpha}\,\bar{\xi}_2\bar{Z}^k, \\ &\langle Z,\xi_1,\xi_2|N,k,1,1\rangle = N(N-1)\sqrt{\alpha(1+\alpha)}\,\bar{\xi}_1\bar{\xi}_2\bar{Z}^k. \end{aligned} \tag{18}$$

While orthogonality is a convenient property for a set of basis vectors it is not a necessary one. The essential property of such a set is that it be complete. Since the $4N$ state vectors $\{|N,k,\alpha_1,\alpha_2\rangle\}$ of an irrep of the spl(2,1) superalgebra are known to form a completeness orthogonal set, the one-parameter supercoherent state $|Z,\xi_1,\xi_2\rangle$ for the spl(2,1) superalgebra can be shown without difficulty to form a complete set. To give a proof we need only demonstrate that the unit operator may be expressed as a suitable sum or an integral, over the superplane, of projection operators of the form $|Z,\xi_1,\xi_2\rangle\langle Z,\xi_1,\xi_2|$. In order to describe such integral we introduce generally the differential element of weight area in the superplane

$$d^2Zd^2\xi_1 d^2\xi_2\sigma(Z,\xi_1,\xi_2)=|Z|d|Z|d\theta d\bar{\xi}_1 d\xi_1 d\bar{\xi}_2 d\xi_2\sigma(Z,\xi_1,\xi_2) \tag{19}$$

where $\sigma(Z,\xi_1,\xi_2)$ is a weight superfield function, and $Z=|Z|e^{i\theta}$.

The problem here may by changed to find the weight superfield function$\sigma(Z,\xi_1,\xi_2)$ such that

$$\begin{aligned}&\int d^2Zd^2\xi_1 d^2\xi_2\sigma(Z,\xi_1,\xi_2)|Z,\xi_1,\xi_2\rangle\langle Z,\xi_1,\xi_2|\\ &=\sum_{k=0}^{N}\binom{N}{k}|N,k,0,0\rangle\langle N,k,0,0|\\ &\quad+\sum_{k=0}^{N-1}\binom{N-1}{k}|N,k,0,1\rangle\langle N,k,0,1|\\ &\quad+\sum_{k=0}^{N-1}\binom{N-1}{k}|N,k,1,0\rangle\langle N,k,1,0|\\ &\quad+\sum_{k=0}^{N-2}\binom{N-2}{k}|N,k,1,1\rangle\langle N,k,1,1|=1,\end{aligned} \tag{20}$$

where $d^2Z=|Z|d|Z|d\theta$, $d^2\xi_1=d\bar{\xi}_1 d\xi_1$, $d^2\xi_2=d\bar{\xi}_2 d\xi_2$.

To determine $\sigma(Z,\xi_1,\xi_2)$ we expand $\sigma(Z,\xi_1,\xi_2)$ in ξ_1,ξ_2, and save four effective items for the integral (19), i.e.,

$$\sigma(Z,\xi_1,\xi_2)=A(Z)+B(Z)\bar{\xi}_1\xi_1+C(Z)\bar{\xi}_2\xi_2+D(Z)\bar{\xi}_1\xi_1\bar{\xi}_2\xi_2, \tag{21}$$

where $A(Z)$, $B(Z)$, $C(Z)$ and $D(Z)$ are four expansion coefficients. Substituting the definition of simple coherent state (12) into (20) and integrating over the entire area of the superplane we have

$$\begin{aligned}&\int d^2Zd^2\xi_1 d^2\xi_2\sigma(Z,\xi_1,\xi_2)|Z,\xi_1,\xi_2\rangle\langle Z,\xi_1,\xi_2|\\ &=\int d^2ZD(Z)|Z\rangle_{1\,1}\langle Z|+N^2(1+\alpha)\int d^2ZC(Z)|Z\rangle_{2\,2}\langle Z|\\ &\quad+N^2\alpha\int d^2ZB(Z)|Z\rangle_{3\,3}\langle Z|\\ &\quad+N^2(N-1)^2\alpha(1+\alpha)\int d^2ZA(Z)|Z\rangle_{4\,4}\langle Z|\\ &=2\pi\sum_{n=0}^{N}\binom{N}{n}\binom{N}{n}\int_0^\infty D(Z)|Z|^{2n+1}d|Z||N,n,0,0\rangle\langle N,n,0,0|\\ &\quad+2\pi\sum_{n=0}^{N-1}N^2(1+\alpha)\binom{N-1}{n}\binom{N-1}{n}\end{aligned}$$

$$\times \int_0^\infty C(Z)\,|Z|^{2n+1}\,d\,|Z|\,|N,n,0,1\rangle\,\langle N,n,0,1|$$
$$+ 2\pi \sum_{n=0}^{N-1} N^2\alpha \binom{N-1}{n}\binom{N-1}{n} \int_0^\infty B(Z)|Z|^{2n+1} d|Z||N,n,1,0\rangle\langle N,n,1,0|$$
$$+ 2\pi \sum_{n=0}^{N-2} N^2(N-1)^2\alpha(1+\alpha)\binom{N-2}{n}\binom{N-2}{n}$$
$$\times \int_0^\infty A(Z)|Z|^{2n+1} d|Z||N,n,1,1\rangle\langle N,n,1,1|. \tag{22}$$

In calculating the integral (22) we have used the following Grassmann integral,

$$\begin{aligned} \int d\xi_1 = \int d\bar{\xi}_1 = \int d\xi_2 = \int d\bar{\xi}_2 = 0, \\ \int \xi_1 d\xi_1 = \int \bar{\xi}_1 d\bar{\xi}_1 = \int \xi_2 d\xi_2 = \int \bar{\xi}_2 d\bar{\xi}_2 = 1. \end{aligned} \tag{23}$$

Comparing (22) with (6) we must have

$$\begin{aligned} 2\pi \binom{N}{n} \int_0^\infty D(Z)|Z|^{2n+1} d|Z| = 1, \\ 2\pi N^2(1+\alpha)\binom{N-1}{n} \int_0^\infty C(Z)|Z|^{2n+1} d|Z| = 1, \\ 2\pi N^2\alpha \binom{N-1}{n} \int_0^\infty B(Z)|Z|^{2n+1} d|Z| = 1, \\ 2\pi N^2(N-1)^2\alpha(1+\alpha)\binom{N-2}{n} \int_0^\infty A(Z)|Z|^{2n+1} d|Z| = 1. \end{aligned} \tag{24}$$

With the aid of the following integral identity (25)

$$\int_0^\infty \frac{x^{2n+1}}{(1+x^2)^m} dx = \frac{n!(m-n-2)!}{2(m-1)!} \tag{25}$$

and by comparing (24) with (25) we obtain the following expansion coefficients

$$\begin{aligned} D(Z) &= \frac{N+1}{\pi(1+\bar{Z}Z)^{N+2}}, \\ C(Z) &= \frac{1}{\pi N(1+\alpha)(1+\bar{Z}Z)^{N+1}}, \\ B(Z) &= \frac{1}{\pi N\alpha(1+\bar{Z}Z)^{N+1}}, \\ A(Z) &= \frac{1}{\pi N^2(N-1)\alpha(1+\alpha)(1+\bar{Z}Z)^{N}}. \end{aligned} \tag{26}$$

Substituting above expansion coefficients into (21) we finally obtain the weight superfield function

$$\sigma(Z,\xi_1,\xi_2)=\frac{1}{\pi}\left[(N+1)\bar{\xi}_1\xi_1\bar{\xi}_2\xi_2+\left(\frac{1}{N\alpha}\bar{\xi}_1\xi_1+\frac{1}{N(\alpha+1)}\bar{\xi}_2\xi_2\right)(1+\bar{Z}Z)\right.$$
$$\left.+\frac{4}{N^2(N-1)\alpha(1+\alpha)}(1+\bar{Z}Z)^2\right](1+\bar{Z}Z)^{-N-2}.$$

We have thus shown

$$\frac{1}{\pi}\int d^2Zd^2\xi_1d^2\xi_2\left[(N+1)\bar{\xi}_1\xi_1\bar{\xi}_2\xi_2+\left(\frac{1}{N\alpha}\bar{\xi}_1\xi_1+\frac{1}{N(\alpha+1)}\bar{\xi}_2\xi_2\right)(1+\bar{Z}Z)\right.$$
$$\left.+\frac{4}{N^2(N-1)\alpha(1+\alpha)}(1+\bar{Z}Z)^2\right](1+\bar{Z}Z)^{-N-2}|Z,\xi_1,\xi_2\rangle\langle Z,\xi_1,\xi_2|=1 \quad (27)$$

which is a completeness relation for the one-parameter supercoherent state of the spl(2,1) superalgebra of precisely the type desired. As a result of the above completeness relation, an arbitrary vector $|\Psi\rangle$can be expanded in terms of the supercoherent state for the spl(2,1) superalgebra. To secure the expansion of $|\Psi\rangle$ in terms of the supercoherent state $|Z,\xi_1,\xi_2\rangle$, we multiply $|\Psi\rangle$ by the representation (27) of the unit operator. We then find

$$|\Psi\rangle=\frac{1}{\pi}\int d^2Zd^2\xi_1d^2\xi_2\left[(N+1)\bar{\xi}_1\xi_1\bar{\xi}_2\xi_2\right.$$
$$+\left(\frac{1}{N\alpha}\bar{\xi}_1\xi_1+\frac{1}{N(\alpha+1)}\bar{\xi}_2\xi_2\right)(1+\bar{Z}Z)$$
$$\left.+\frac{4}{N^2(N-1)\alpha(1+\alpha)}(1+\bar{Z}Z)^2\right](1+\bar{Z}Z)^{-N-2}$$
$$\times|Z,\xi_1,\xi_2\rangle\langle Z,\xi_1,\xi_2||\Psi\rangle. \quad (28)$$

4 Conclusion

We have constructed one-parameter supercoherent state of the spl(2,1) superalgebra. We have discussed the orthogonality and completeness relations for the supercoherent state of the spl(2,1) superalgebra. On the basis, we can study new one-parameter inhomogeneous differential realizations of the spl(2,1) superalgebra.

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